Patent Application of

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for

TITLE: SPHERICAL AND POLYHEDRAL SHELLS WITH IMPROVED SEGMENTATION

BACKGROUND OF THE INVENTION—Field

The present invention relates to shells having spherical curvature, or generally spherical form, including polyhedral domes.

—Prior Art

A sphere encloses the maximum volume within a given surface area, and offers great potential for conservation of energy and material resources. Replacement of rectangular and cylindrical structures by spherical shells can reduce heat transfer loss 25%, and can reduce structural material requirements by 50%. Spherical shells have important applications in architecture and for light-weight pressurized tanks. Despite their many potential benefits, shells of generally spherical form have been difficult to manufacture and assemble.

As used here, a shell of generally spherical form has the appearance of a sphere or portion of a sphere, a portion of a polyhedron having an approximately spherical shape, or any other group of polygons joined together in such fashion, including geodesic domes. A common way to manufacture large spherical shells is to join together small segments to form a whole shell. Some examples are found in architecture, as electronic antenna enclosures, and in manufactured pressure tanks. In the mid-twentieth century, Richard Buckminster Fuller

popularized the use of sphere-like polyhedra in architecture, called "geodesic domes." U. S. patents 2,682,235 (1954), 2,881,717 (1959), 2,914,074 (1959), 3,197,927 (1965), and 3,203,144 (1965), all to Fuller, disclose the use of many-faced polyhedra as models for dome structures. Fuller divided the faces of a regular polyhedron into groups of smaller triangles; the resulting polyhedra approached a spherical shape as the number of triangular faces increased. Each of Fuller's dome segments consisted of one triangular face, but a segment may comprise several faces, only part of a face, or any other fraction of the whole.

Spherical and polyhedral shells or domes may be divided into two structural types: 1) shells having a skin or membrane only, and 2) "braced domes" which are composed of a skin and a supporting framework. Fuller taught that the most efficient structural design is a dome with a frame composed of a triangular or tetrahedral lattice. Fuller's domes are often built by attaching segments of an outer membrane to a strong frame of struts and connecting hubs conforming to the edges and vertices of a many-faced polyhedron. Construction from segments is especially useful for large sphere-like structures manufactured at one location and then shipped to a second location for assembly.

Spherical shells offer great structural advantages. It is well known that the skin of a spherically-shaped building can serve as its own structural support so that no bracing framework is required. Monolithic domes are notably built on the intended site, as one piece, with no supporting frame. The method is used by the Monolithic Dome Institute of Italy, Texas. Hardening materials are sprayed onto a pre-assembled steel reinforcing backed by an inflated spherically-shaped form. There are difficulties in construction of monolithic domes and also in making later modifications. Spherical shells made from segments have lower cost and are easier to build or modify.

The design of segments for spherical shells and domes in the prior art will now be described in order to provide a background for the invention. A segment is meant to be a surficial portion of a shell, as contrasted to an underlying structural portion, although structural elements or capabilities may be incorporated within a physical segment. Physical segments designed for assembly of a sphere or dome have the same form as geometric segments obtained by division of the equivalent geometric figure, but may have additional features, such as attachment flanges for joining them together. For the present purpose, a geometric segment is any desired portion of a geometric figure and is a mathematical construct, while the physical

segment is a useful article having the same general shape. In many instances I will simply refer to a segment and the reader will be able to determine the type of segment. The basic principles of designing segments will be illustrated by discussion of geometric divisions of spheres and regular polyhedra. Additional features of physical segments will be taught by later reference to specific examples and embodiments.

—Division of Spheres and Polyhedra

Regular polyhedra can be used to divide a sphere into equal parts with a high degree of symmetry. For example, consider a cube inscribed in a sphere so that the central points of the sphere and cube coincide and all corners of the cube lie on the spherical surface. If the edges of the cube are projected onto the sphere by tracing radii that pass through the cube's edges, then great circle arcs are formed on the sphere that divide it into six equal parts, each part being a spherical square. The resulting spherical cube is illustrated in Fig. 1A. Similarly, the projection of any regular polyhedron onto a circumscribed sphere generates a corresponding regular spherical polyhedron with regular spherical polygonal faces. The lines and corners about these faces are referred to as edges and vertices in similarity to ordinary polyhedra.

The derivation of useful polyhedral or spherical polyhedral forms by division of faces of the regular polyhedra will now be summarized. The faces of a regular dodecahedron comprise 12 regular pentagons, each having five-fold rotational symmetry. Consequently, each face may be divided into five identical parts. As taught by Fuller, each pentagon may be divided into five triangles by drawing lines from the center of each pentagon to its corners. If the center point of each pentagon is also raised to the height of a vertex (i.e., lying on the circumscribed sphere), a new polyhedron is formed having 60 identical faces and 32 vertices, as shown in Figs. 1B and 1C. Three divided pentagons, such as in Fig. 1B, are visible in Fig. 1C. This polyhedron is not regular because each triangle has a larger angle at the newly-formed vertex where five triangles join and two smaller angles at the other vertices which are coincident to six triangles. The new faces have no rotational symmetry and cannot be divided further into identical parts. The new polyhedron may be projected onto the circumscribed sphere as previously described to produce a similarly-formed spherical polyhedron having 60 identical spherical triangular faces. As used here, faces are identical only if they can be superposed without resorting to a reflection or to turning over of one of the faces. For the purpose of

obtaining small, useful faces, 60 is the maximum number of identical faces that a polyhedron can have. Alternatively, $4\pi/60 = \pi/15$ steradians is the minimum solid angle that such an identical face can occupy. The rule applies equally well to spherical polyhedra.

In mathematics, the division of surfaces into identical or repeating patterns of parts is referred to as tiling. Tiling, or segmentation, of the sphere has been investigated by many geometers, and artists such as M. C. Escher. Robert Dawson of Saint Mary's University, Halifax, Nova Scotia, has described many tilings of the sphere by isosceles triangles. Some of these tilings divide a sphere into more than 60 identical triangular segments. However, these segments always have a long dimension of at least 60 degrees of arc which severely restricts their usefulness for manufacture of large objects.

The regular icosahedron has 20 identical equilateral triangular faces and 12 identical vertices. Each face has three-fold rotational symmetry and can be divided into three identical parts. The number of identical faces so obtained is 60, as before. However, Fuller abandoned the requirement for identical faces and divided the icosahedron in another way. He typically divided each face into four triangles by drawing lines between the center points of the three sides, forming 30 additional vertices, for a total of 42. The resulting new polyhedron has 80 triangular faces and 120 edges, as shown in Figs. 1D and 1E. The newly-formed triangles are not all alike. The 30 new vertices join six edges (or triangles) while the original 12 vertices join only five. Of each four new triangles, the center triangle has only new vertices while the other three include an old vertex with a larger angle. The center equilateral triangle is somewhat larger than the three isosceles triangles. Fuller typically continued to divide these triangles into yet smaller triangles. Fuller also performed further divisions of the above 60-face polyhedron into smaller triangles, and he referred to the process as "triacon breakdown." The smaller triangles resulting from Fuller's divisions are not identical, having a variety of different sizes and shapes. In this way, the construction of Fuller's geodesic domes becomes increasingly complex and difficult as the domes become larger, and larger numbers of triangular faces are required.

Quadrilaterals, pentagons, and hexagons can also be used for faces of polyhedral structures. A few large dome structures have been built with hexagonal and pentagonal faces. These domes have generally used an underlying triangular framework for structural support. Some shells have been built successfully using quadrilateral segments that are rhombic or

trapezoidal. U. S. patent 4,181,235 to Baysinger (1980) describes the manufacture of spherical tanks from spherical trapezoidal segments. This shell construction is very cumbersome, requiring segments of several sizes and shapes to form the structure.

Oblong quadrilaterals were used by Berger Brothers Company of New Haven, Connecticut, to form an inflated fabric dome. Only a photograph is available, as shown in the book, *The Dymaxion World of Buckminster Fuller* by Robert Marks and R. Buckminster Fuller (Anchor Press/Doubleday, 1973). Their construction was obtained by rearrangement of the faces on one of Fuller's many-faced geodesic domes. Fuller's triangular faces were projected onto a sphere and combined pair-wise to form spherical four-sided figures. Then, these figures were divided in half to form oblong quadrilaterals. While the spherical quadrilaterals so formed were "parallelogram-like" in their appearance, they were distorted and they varied in size and shape because the parent four-sided figures had unequal sides, and varied in size and shape.

The number of segments used for this Berger half dome is 120, which exceeds the limit of 30 for domes having strictly identical segments (except as noted above for certain spherical triangles). Furthermore, Berger Brothers failed to achieve the higher level of uniformity that is possible with this degree of segmentation. As with Fuller's triangles, the Berger segmentation produced elements that were difficult to produce and assemble. In summary, prior efforts to segment the sphere have failed to realize the benefits of having identical, conveniently-shaped segments.

In the prior art, oblong segments having the approximate shape of a parallelogram, and with substantially uniform size and width, have not been used to construct shells of generally spherical form. However, a route to forming such segments is provided by two rhombic polyhedra: the rhombic dodecahedron, with 12 rhombic faces, as shown in Fig. 1F; and the triacontahedron, with 30 rhombic faces, as shown in Fig. 1G. They are termed "half-regular" in that they have identical faces, but the faces have only equal sides and not equal angles.

If these rhombic polyhedra are projected onto a circumscribed sphere, they generate corresponding spherical rhombic polyhedra with spherical rhombic faces. Fig. 1H depicts a spherical triacontahedron. In analogy to plane geometry, the sides of a spherical rhombus are equal and opposite angles are equal. Each face of the spherical triacontahedron has sides of arc length 37.3774 degrees. The acute angles are 72 degrees and the obtuse angles are 120 degrees. The width between the sides of the face at its center is 36 degrees exactly. The face of a

spherical rhombic dodecahedron has corners of 90 degrees and 120 degrees, and the length of its sides is 54.7356 degrees. This spherical rhombus has a width between sides of 60 degrees exactly. In the discussions to follow, only the triacontahedron and its spherical analog are presented, but similar results apply for the rhombic dodecahedron and its spherical form.

In plane geometry, the opposite sides of a rhombus are parallel but, for a spherical rhombus, the concept of parallel sides requires reexamination. In Euclidean geometry, two parallel lines are said to intersect at infinity. The opposite sides of the spherical rhombus intersect at the farthest poles of the sphere, with the center of the rhombus lying on the "equator." Thus, the sides may be viewed as parallel in a spherical sense. They are also practically parallel in the Euclidean sense, varying in separation by only a small amount across a spherical rhombus of the types discussed. The spherical rhombic faces of these half-regular spherical polyhedra have two-fold rotational symmetry and therefore each face may be divided in two to yield twice the number of identical faces. Thus, a division of the spherical triacontahedron can produce a spherical polyhedron with 60 identical faces. In dividing the faces, it is not necessary to form triangles. Any arc passing through the center of a face will divide it into identical parts, either spherical triangles or oblong spherical quadrilaterals. If the dividing line is generally parallel to opposite sides of the spherical rhombus, then the resulting spherical quadrilaterals have the general form of a parallelogram conformed to a sphere. No practical application of division of the half-regular spherical polyhedra into oblong quadrilaterals is known in the prior art. Nor is it known to divide ordinary polyhedra, with polygonal faces, into oblong segments.

Spherical polyhedra have thus far been described in terms of their correspondence to ordinary polyhedra. However, on a spherical polyhedron, if an edge terminates on another edge at a point other than an end point, then there is no corresponding ordinary polyhedron composed of edges and vertices. This non-correspondence will be present with regard to many spherical embodiments of my improved shell segmentation. While the maximum number of identical oblong segments is 60 in the strict mathematical sense (except as previously mentioned), it will be shown in the detailed description that practical constructions of spherical shells may be accomplished with larger numbers of identical segments.

—Limitations and Deficiencies of Present Domes and Spheres

Manufacture of large domes or spheres is comparatively rare. Less than one building in a thousand in the United States has a dome as part of its structure. Since dome-shaped structures offer many benefits, one may conclude that compelling deficiencies limit their use. Presently, most architectural domes are braced domes. I will review some of the disadvantages of braced-dome structures, using the example of geodesic domes for illustration. The foremost reason for trepidation in building geodesic domes is their inherent complexity. A typical geodesic dome house is made of triangles having several different sets of dimensions. Likewise, struts and hubs for the bracing framework are made in several sizes and shapes according to their location in the dome. The varying triangular pieces, struts, and hubs must have precise dimensions and be assembled in precisely the right relationships. In most instances, the dome components are pre-cut in a factory to minimize problems by the builder. In addition, the pieces are typically coded to minimize confusion during assembly. For the usual geodesic dome, difficulties in assembly of the shell are only the beginning.

Construction of floors, walls, and other architectural elements involves fitting them firmly to the irregular surfaces of the geodesic dome. Each wall stud and floor joist contacting the dome's interior surface will have a unique length and best angle of cutoff. Typically, the interior surface of a geodesic dome is made of a strong material, such as oriented strand board, in order to secure a firm attachment at the end of each floor and wall element that contacts it. Nearly every edge and corner in the lower portion of the polyhedral structure presents the need for more material cuts in virtually unpredictable directions and dimensions, resulting in excessive waste of material and time. Because of the frequent delays and high costs experienced with building of dome houses, geodesic domes never find their way into the rapidly built and cost-efficient tract housing developments. Other difficulties are that the cutting of the triangular dome faces from rectangular stock can be wasteful, and that the faces are typically too large to cut from standard four-foot-wide material. This, combined with their angularity and variations in size and shape, contribute further to problems in manufacture and shipping of the dome faces.

Most architectural domes are built with a bracing framework. In most instances, braced domes are designed without regard for any structural contribution from the skin materials. Nevertheless, the flat face panels must be of sufficient strength and stiffness to prevent flexure

under static loading, and vibration in dynamic loading such as wind gusts. The particular structural requirements of polyhedral domes thus require use of premium materials that are more expensive than the materials used in normal construction.

There are additional problems related to the frames used in geodesic domes. The face panels of a braced dome are attached to the frame along their perimeters and meet one another at an angle along polyhedral edges. Such an arrangement of parts is difficult to assemble due to combined effects of small dimensional variations and the requirement that pre-cut panels must fit within their spaces. Consequently, considerable space is allowed between panels to insure that they will fit. The gapped edge connections are poorly suited to provide hermetic sealing and to transfer structural loads. Further, the gaps lie over the underlying framework and its connections which are then vulnerable to attack by the elements. Typically, geodesic dome houses require a covering of shingles to help protect them from moisture. At every vertex of a geodesic dome, the frame struts are typically joined together by steel hubs with bolted connections. As geodesic domes have vertices that are not all identical, multiple hub types are required for the assembly. These special hardware parts are different for the various polyhedral designs. Thus, considerable cost is associated with these parts. Lastly, the frame members, edges and vertices are placed in the structure with only secondary regard for the designer's plan for use of the structure, such as for placement of doors and windows.

True spherical shells can overcome many of the above problems in dome architecture. But, a problem with spherical shells has been the lack of a convenient way to manufacture the segments and easily assemble them. This problem has also been an important factor in limiting use of spherical tanks for containing pressurized fluids. A common method used in the past for assembly of true spheres and spherical domes from segments depends upon division of a sphere by longitudinal lines, obtaining very long, lune-shaped segments. In this case, all segments are identical in form but have an awkward size and shape. Another common method depends upon division of a sphere according to both longitudinal and latitudinal lines. In this second instance, the segments are of convenient sizes but have several different sizes and shapes. U. S. Patent 3,945,236 to Hooper (1976) illustrates this method. Similarly awkward segmentation schemes are shown in U. S. patents 4,181,235 to Baysinger (1980) and 5,697,312 to Gustaffson et al. (1997). All of the methods presently in use for construction of spherical shells lack the ease of

manufacture, shipment, and assembly, and other benefits, that could be obtained by having identical, conveniently-shaped segments.

It is evident that the usefulness of spherical and polyhedral structures or products may be substantially advanced by overcoming a number of limitations and deficiencies associated with their design and manufacture.

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SUMMARY

According to the present invention, spherical and polyhedral shells are segmented in a manner that substantially improves cost and efficiency of manufacture. The physical segments are modeled after small, conveniently-shaped, oblong segments of geometric figures. The wide range of applications will be illustrated by a few representative examples only.

Segments of a sphere are constructed by dividing a face of a spherical triacontahedron into quadrilateral segments, using lines running generally parallel to the sides of the face. In a preferred embodiment, one line is used to divide the spherical rhombic face into two identical oblong segments. This geometric segment form serves as a model for manufacture of physical segments that are useful for construction of spheres and domes in a variety of applications. Examples are pressurized storage tanks, light-weight greenhouses, and skylights. In other embodiments, my improved segmentations are used for assembly of spherical shells from any number of convenient oblong quadrilateral parts, and for manufacture of various polyhedral shells. The segments may include parts for convenient assembly or structural support

—Objects and Advantages

Accordingly, an object of the present invention is to provide improved shells of generally spherical form, having substantially simplified construction. A second object is the conservation of energy and material resources by application of improved shells. Other objects and advantages are:

1. Use of oblong, substantially identical segments having readily manufactured form.
2. Use of small segments having a convenient shape for packing and shipping.
3. Elimination of bracing frameworks and associated manufacturing difficulties.
4. Reduction of labor and materials costs, and material wastage.

5. Assembly of truly spherical shells for improved weight and strength.
6. Connections with better load transfer and less precise dimensional requirements.
7. Wide manufacturability in many materials and environments.

Further objects and advantages will become apparent from a consideration of the ensuing description and drawings.

DRAWINGS—Figures

The previously referenced prior-art figures (and figures generally known from solid geometry) are labeled as Figs. 1A through 1H, as follows:

FIG. 1A depicts a spherical cube, as generally known from solid geometry.

FIG. 1B shows a division of a dodecahedron face, as known in the prior art.

FIG. 1C depicts the polyhedron formed by the division operation shown in FIG. 1B, as known in the prior art.

FIG. 1D illustrates a division of an icosahedron face, as known in the prior art.

FIG. 1E depicts a geodesic dome resulting from the division operation illustrated in FIG. 1D, as known in the prior art.

FIGS. 1F and 1G illustrate the two half-regular polyhedra, as generally known.

FIG. 1H depicts a spherical triacontahedron, as generally known from solid geometry.

In the following drawings of the invention, the figures are labeled numerically with closely related figures shown by alphabetic suffixes, as follows:

FIG. 2A is a sphere divided into oblong segments in the simplest possible form.

FIG. 2B illustrates a half-spherical dome according to the design of FIG. 2A.

FIGS. 3A, B illustrate two types of the simplest-form oblong segments.

FIG. 3C is an enlarged segment of FIG. 3A form showing more detail.

FIGS. 4A, B show a segment of FIG. 3A form with added overlapping attachment flanges.

FIG. 4C shows an overlapping connection of the above flange in cross-sectional view.

FIG. 5A shows an insulated, double-shell segment version of the FIG. 3A form.

FIG. 5B shows the segment of FIG. 5A in cross-sectional view.

FIG. 6 depicts the assembly of a 1/30th part of a sphere from four identical segments.

FIG. 7 exemplifies a class of further ramifications of division of spherical shells.

FIG. 8 shows a 1/15th portion of a geodesic dome with segments of the FIG. 3A form.

FIG. 9 is a similar 1/15th part of a polyhedral dome having 240 rhombic faces.

FIG. 10 is a 1/15th portion of a polyhedral dome with two groups of identical segments.

FIG. 11A is a perspective view of part of a polyhedron having 132 polygonal faces.

FIG. 11B is a further ramification, dividing the polyhedron of FIG. 11A into identical parts.

FIG. 12 depicts a further ramification having oblong segments and interstitial elements.

FIG. 13 depicts a substantially square-shaped spherical shell having oblong segments.

—Reference Numerals

20	sphere in Fig. 2A
22	great circle line of Fig. 2A and bottom edge of Fig. 2B
24	segment portion (small part) of Fig. 2B
25	segment portion (large part) of Fig. 2B
26	cut-off segment portion in Fig. 2A
27	remaining partial segment in Fig. 2A
28	partial segment in Fig. 2A
29	segment portion in Fig. 2A
30	segment (basic), and mirror image of segment 32 of Fig. 3B
32	segment (basic) of Fig. 3B, and mirror image of segment 30
34	side of spherical rhombic surface in Fig. 3C (first)
36	side of spherical rhombic surface in Fig. 3C (second)
38	side of spherical rhombic surface in Fig. 3C (third)
40	line of division (basic) and first long side of segments in Figs. 3A-3C
41	line perpendicular to both lines 38 and 42 in Fig. 3C
42	side of spherical rhombic surface in Fig. 3C (fourth)
44	first short side in Figs. 3A-3C
46	second short side in Figs. 3A-3C
48	first angle in Figs. 3A-3C
50	second angle in Figs. 3A-3C

52 third angle in Figs. 3A-3C

54 fourth angle in Figs. 3A-3C

56 flange, corresponding part of adjoining segment (upper) in Fig. 4B

58 flange, corresponding part of adjoining segment (lower) in Fig. 4B

60 physical quadrilateral segment of Figs. 4A and 4B

62 flange, extends outwardly from lower half of first long side

64 flange, extends from (upper) short side, and underlapping flange portion of Fig. 4C

66 flange, extends from (lower) short side of Figs. 4A and 4B

68 flange, same part (as 62) of mating segment of Fig. 4B

70 overlapping edge of an adjoining segment in Fig. 4C

72 fastener in Fig. 4C

74 optional adhesive placed along and between faying surfaces in Fig. 4C

76 exposed surface near overlapping line (segment 60) in Fig. 4C

77 overlapping line in Fig. 4C

78 exposed surface near overlapping line (mating segment) in Fig. 4C

80 insulated segment of Figs. 5A and 5B

84 inner shell segment in Fig. 5B

86 outer shell segment in Fig. 5B

87 short splicing-offset element in Fig. 5A

88 long splicing-offset element in Figs. 5A and 5B

90 insulation in Figs. 5A and 5B

92 female connector portion (first), left and top of Fig. 5A, and in Fig. 5B

93 female connector portion (second), at bottom of Fig. 5A

94 edge of outer shell segment 86 in Fig. 5B

96 edge of inner shell segment 84 in Fig. 5B

98 spacer in Fig. 5A

99 small void in Fig. 5A

100 spherical rhombic face assembled from four segments in Fig. 6

102 one of four segments of uniform crosswise section in Fig. 6

108 one of 36 segments in Fig. 7

110 spherical rhombus having 36 segments per 1/30th part of a sphere in Fig. 7

112 one of eight triangular faces in Fig. 8

114 line joining with line 116 to form first long side of oblong segments in Fig. 8

116 line joining with line 114 to form first long side of oblong segments in Fig. 8

118 center vertex in Fig. 8

120 segment, first of two equal parts formed by lines 114 and 116 in Fig. 8

122 line joining with line 124 to form second long side of segment 120 in Fig. 8

124 line joining with line 122 to form second long side of segment 120 in Fig. 8

126 first short side of segment 120 in Fig. 8

128 second short side of segment 120 in Fig. 8

130 segment, second of two equal parts formed by lines 114 and 116 in Fig. 8

132 line joining with line 134 to form the second long side of segment 130 in Fig. 8

134 line joining with line 132 to form the second long side of segment 130 in Fig. 8

136 first short side of segment 130 in Fig. 8

138 second short side of segment 130 in Fig. 8

140 segment comprising the right half of Fig. 9

142 rhombic face included in segment 140 of Fig. 9

150 middle segment of Fig. 10

154 corners of outside segments with interior angles measuring 72° in Fig. 10

156 corners of outside segments with interior angles measuring 120° in Fig. 10

158 half hexagon on top end of middle segment 150 in Fig. 10

160 side segment (two) in Fig. 10

162 half hexagon on bottom end of middle segment 150 in Fig. 10

164 half hexagons of side segments 160 in Fig. 10

170 segment of Fig. 11B

172 triangular tip, portion of segment 170 in Fig. 11B

174 first hexagon face of Figs. 11A and 11B

176 second hexagon face of Figs. 11A and 11B

178 pentagon face of Figs.11A and 11B

180 quadrilateral segment, one of the oblong faces of Fig. 12

182 first long side of quadrilateral 180 in Fig. 12

184 second long side of quadrilateral 180 in Fig. 12

186	first short side of quadrilateral 180 in Fig. 12
188	second short side of quadrilateral 180 in Fig. 12
190	first angle of quadrilateral 180 in Fig. 12
192	second angle of quadrilateral 180 in Fig. 12
194	third angle of quadrilateral 180 in Fig. 12
196	fourth angle of quadrilateral 180 in Fig. 12
198	spherical pentagon, one of the interstitial elements of Fig. 12
200	shell
202	square group, one-fourth part of shell 200 in Fig. 13
204	oblong segment in Fig. 13
P	center point of the spherical rhombic surface
Wt	top width of segment 30
Wb	bottom width of segment 30
Wm	right half of line 41 and middle width of segment 30

Definitions

In the present patent, the following geometrical and engineering terms have the definitions indicated.

Braced dome.—A dome having structural members for strength in addition to its skin or laminar structure.

Center of symmetry.—A point about which a body is symmetric. Surfaces or planar figures with rotational symmetry have a center of symmetry, at the center of rotation. The center of a regular polygon is a center of symmetry.

Central point.—The average of all points in a figure. The center of symmetry for a regular polyhedron.

Circumscribed sphere.—The smallest sphere that will enclose a solid figure. The farthest points of the figure (at least two) lie on the surface of the circumscribed sphere.

Coincide.—To be at a common place. If two points are *coincident*, then they are actually the same point.

Conformed.—Having become the same in form; projected onto.

Degrees.—Units of measure for angles, lines on spheres, and sides of spherical segments.

Division of a polyhedron.—A process introduced by R. Buckminster Fuller to increase the number of faces of a polyhedron, to cause it to be more spherical. Each face of a polyhedron is divided into smaller triangles, and newly-formed vertices are displaced outward to touch a circumscribed sphere. Examples are shown in prior art Figs. 1B and 1D.

Division of a sphere.—A process in which edges and vertices of a polyhedron are projected onto a circumscribed sphere. The projected lines and points on the sphere create a corresponding spherical polyhedron with spherical polygonal faces. The lines and corners about the spherical faces are referred to as edges and vertices in similarity to their names in ordinary polyhedra. The spherical polyhedron can be further divided by placing additional lines within the faces.

Dodecahedron.—A polyhedron having twelve faces. A dodecahedron with identical regular pentagon faces is a regular dodecahedron, one of only five regular polyhedra.

Dome.—A shell having spherical curvature or generally spherical form, modeled after a top portion of a sphere or sphere-like polyhedron. It is usually a portion of a building structure.

Edge of a polyhedron.—The common side between two faces, terminating at two vertices.

Edge of a spherical polyhedron.—The common side between two spherical polygonal faces, terminating either at vertices or sides. A dividing line between faces.

Euclidean geometry.—A geometry concerning straight lines and planes in three dimensions, as distinct from spherical geometry.

Face of a polyhedron.—A polygon of the polyhedron.

Face of a spherical polyhedron.—A spherical polygon.

Face of a shell or dome.—The flat part of a shell or dome corresponding to a face of the polyhedron after which it is modeled.

Generally spherical form.—The form of a sphere; includes bodies having the form of a portion of a sphere; also includes polyhedral bodies that appear to be substantially spherical. A term originally introduced by R. Buckminster Fuller in his U. S. patent 2,682,235 (1954). (See his Claim 1.) My Figs. 1A, 1C, 1E, 1G, 1H, 2A, 2B, and 11A are illustrative. See *spherical.*

Geodesic dome.—In current usage, a polyhedral dome having a generally spherical form.

Geometric figure.—In mathematics, a construct composed of lines and surfaces.

Geometric segment.—A portion of a geometric figure.

Great circle.—A circle formed on a sphere by the intersection of the sphere and a plane passing through its center. A portion of a great circle is a *great circle arc.*

Half-regular polyhedron.—A polyhedron in which all of the faces are identical and all of the edges are equal, but the angles are not all equal. There are only two such figures of interest here; the rhombic dodecahedron with twelve identical rhombic faces (shown in prior art Fig. 1F), and the triacontahedron with thirty identical rhombic faces (shown in prior art Fig. 1G).

Half-regular spherical polyhedron.—The figure obtained by projecting a half-regular polyhedron onto the circumscribed sphere. An example is shown in Fig. 1H.

Icosahedron.—A polyhedron having twenty faces. An icosahedron whose faces are identical equilateral triangles is a regular icosahedron, one of only five regular polyhedra.

Identical.—Faces are identical only if they can be moved into exact superposition (superposed) without resorting to turning over of one of the faces, or to a reflection. Physical or geometric segments are identical if they satisfy this same relationship.

Infinity.—A point on a line at an infinite distance from the point of reference. The point at which parallel lines intersect.

Inscribe in (a sphere).—To place in a circumscribed sphere.

Isosceles triangle.—A triangle with two equal sides and two equal angles.

Jointed line.—A line composed of straight segments connected end-to-end.

Model, mathematical.—A mathematical construct that imitates a physical reality. A sphere is a model of a ball. One may also say that a ball is modeled after a sphere.

Octahedron.—A polyhedron having eight faces. An octahedron whose faces are identical equilateral triangles is a regular octahedron, one of only five regular polyhedra.

Operation.—A process that changes a figure into another figure. Division of a polyhedron is an operation introduced by R. Buckminster Fuller. Rotation is a symmetry operation.

Parallel.—Two lines *l* and *m* are parallel if they lie in the same plane and no point (except a point at infinity) lies on both lines. Two great circle lines *l* and *m* on a sphere are parallel at the midway line (equator) between their two points of intersection (poles).

Parallelogram.—A quadrilateral in the plane in which opposite sides are parallel and equal, from which it follows that opposite angles are equal. An oblong segment of a spherical or polyhedral shell that approximates these conditions has a "generally parallelogram shape."

Physical segment.—A portion of a physical object such as a shell.

Polygon.—A simple closed curve in the plane, made up of line segments, or sides.

Polyhedron.—A solid bounded by planes. Its faces are polygons. Plural *polyhedra.* Polyhedra considered here are those having a generally spherical form, such as the icosahedron.

Polyhedral.—Having many faces, as in a polyhedron. Not spherical.

Projection.—A mapping of one surface onto another, in the fashion of a shadow or image cast by a point of light. A figure made by projection. Collapsing of one dimension of an object in Cartesian space is a linear projection, equivalent to having the point of light at infinity.

Quadrilateral.—A figure with four sides.

Reflection.—Reversal of the elements of an object through a plane.

Regular icosahedron.—A polyhedron with twenty identical equilateral triangular faces, one of only five regular polyhedra.

Regular polygon.—A polygon having all sides equal and all angles equal.

Regular polyhedron.—A polyhedron whose faces are identical regular polygons. The five regular polyhedra are: the tetrahedron, cube, octahedron, dodecahedron, and icosahedron.

Regular spherical polygon.—A spherical polygon with all sides equal and all angles equal.

Regular spherical polyhedron.—A projection of a regular polyhedron onto a sphere.

Rhombic dodecahedron.—A polyhedron having twelve rhombic faces. The rhombic dodecahedron (with identical faces) is one of only two half-regular polyhedra. See Fig. 1F.

Rhombic polyhedron.—A polyhedron whose faces are rhombi.

Rhombus.—A quadrilateral in a plane with all sides equal. Plural *rhombi*, adj. *rhombic.*

Rotational symmetry.—The property of invariance under a rotation. There are three rotations of an equilateral triangle about its center, in the plane, of 120°, 240°, and 360°, which leave the triangle as it was, or invariant. Thus, the equilateral triangle has a three-fold rotational symmetry for this axis, or a three-fold axis of symmetry.

Segment, geometric.—A portion of a geometric figure.

Segment, physical.—A portion of a physical object, such as a shell or dome.

Segmentation.—The process of dividing into segments. Ideally, division into one or more groups of identical segments having small, useful, or convenient shapes.

Shell.—A shell is the outer, enclosing portion of a physical structure. Examples are the roof and outer walls of a house, a tank for holding fluids, or generally any outer covering. The

shells considered here have the appearance of a part of a sphere (or whole sphere), a portion of a sphere-like polyhedron, or of any other similar group of polygons.

Sphere-like polyhedron.—A polyhedron similar in appearance to a sphere. The 60-face polyhedron shown in prior art Fig. 1C is an example.

Spherical.—Having the form of a sphere or one of its segments. Relating to or dealing with a sphere or its properties. (*Merriam-Webster Dictionary*, CD Edition, 2002.)

Spherical curvature.—Curvature in two directions with a common center, as in a sphere.

Spherical geometry.—The study of geometry on the sphere. Lines are great circles, and spherical figures have great circle arcs as sides. Lengths and angles are measured in degrees or radians. Area is measured as the subtended solid angle, in steradians.

Spherical icosahedron.—A spherical polyhedron with twenty faces. A regular spherical icosahedron has faces that are identical equilateral spherical triangles.

Spherical polygon.—A many-sided figure on a sphere. The sides are great circle arcs.

Spherical polyhedron.—A sphere divided into faces by great circles or great circle arcs.

Spherical quadrilateral.—A spherical polygon with four sides.

Spherical rectangle.—A spherical quadrilateral with four equal angles. Opposite sides are equal and parallel. The angles are more than 90°.

Spherical rhombic face.—A spherical-rhombus face of a spherical polyhedron.

Spherical rhombic polyhedron.—A spherical polyhedron having only spherical rhombic faces. A spherical triacontahedron is a spherical rhombic polyhedron with thirty faces.

Spherical rhombus.—A spherical quadrilateral with equal sides, and opposite angles equal.

Spherical shell.—A shell with spherical curvature. A shell of spherical form.

Spherical square.—A regular spherical polygon with four sides.

Spherical trapezoid.—A spherical quadrilateral with only two sides parallel. Also used loosely to describe a similar portion of a sphere bounded by latitudinal and longitudinal lines.

Spherical triangle.—A simple closed curve on the sphere composed of three great circle arcs. A spherical polygon with three sides.

Spherically-shaped.—Having the shape of a sphere or part of a sphere. Having spherical curvature.

Spheroidal.—Having the shape of a spheroid, or a symmetric portion of a spheroid. A spheroid is a body formed by revolving an ellipse about its major or minor diameter. An oblate spheroid, formed by revolving about a minor diameter, is a useful shape for a skylight.

Steradian.—A unit of measurement for solid angles, and also for area on a sphere. The total solid angle about a point is 4π steradians. A sphere has 4π steradians of spherical area.

Superposition.—Placing one figure upon another so that all like parts coincide.

Symmetry.—The property of invariance under a symmetry operation.

Tetrahedral lattice.—A network of tetrahedra extending in two or three dimensions.

Tetrahedron.—A polyhedron with four faces. If the faces are identical equilateral triangles, then it is a regular tetrahedron, one of only five regular polyhedra. Plural *tetrahedra.*

Trapezoid.—A quadrilateral having exactly two parallel sides.

Triacon breakdown.—The process of forming a many-sided polyhedron by division of a triacontahedron. The term was coined by R. Buckminster Fuller.

Triacontahedron.—A polyhedron having thirty rhombic faces. The triacontahedron (with identical faces) is one of only two half-regular polyhedra. See Fig. 1G.

Triangular lattice.—A two-dimensional network of connected triangles. The frame of a geodesic dome is a triangular lattice, or sometimes a tetrahedral lattice.

Vertex.—A point at which sides or edges join. The number of edges joining at a vertex of a polyhedron is three or more. A spherical polyhedron has a second type of junction in which an edge terminates on another edge at a point other than an end. Plural *vertices.*

Width of a segment. The meaning varies somewhat, depending upon the application, as indicated by the following instances.

WIDTH OF AN OBLONG SEGMENT.—Determine dimensions in the segment by projection onto an X-Y plane, chosen as follows. Place X in the direction of the segment's greatest extent (end-to-end, and generally parallel to the long sides): x gives position along the length of the segment. Place Y and the origin O so that $w = |y_2 - y_1|$ is as large as possible at $x = 0$. This maximum value is w_O. y_1 and y_2 are functions of x which define opposite edges of the segment. Then, w is the width of the segment and is a function of x. The maximum width of the segment, w_O, would be used for a box that holds the segment. A segment of uniform width

has a constant w for most its length. If two segments A and B are identical, then $w_A = w_B$ at every position along their length, but w_A and w_B need not be constant.

WIDTH OF A SPHERICAL SEGMENT.—An arc in the spherical surface terminating at the points defined by (x, y_1) and (x, y_2) in the plane. The measure of the arc in degrees.

WIDTH OF OTHER THAN AN OBLONG SEGMENT.—The maximum width is the middle dimension of the rectangular box of least volume that is able to hold the segment.

DETAILED DESCRIPTION—Basic Form: Spherical Shell with 60 Oblong Segments —FIGS. 2A to 3C

A first embodiment, representing my improved shell segmentation, is illustrated in Figs. 2A to 3C, and detailed by numeric examples in Table 1, and with accompanying descriptions. This is my improved shell segmentation in its simplest possible form, utilizing an oblong quadrilateral, 1/60th segment of a sphere without appurtenances. Several variations of the basic form will be described.

Fig. 2A shows a sphere 20 divided into 60 identical oblong segments, such as segment 30. These segments are obtained by dividing in two the spherical rhombic faces of a spherical triacontahedron, which was previously described and illustrated in Fig. 1H. The lines of division, such as line 40, are great circle arcs and are placed midway between and generally parallel to two sides, thus forming 60 identical quadrilateral segments of nearly uniform width. This is the maximum number of small, useful segments that can be identical. Having the maximum number of identical segments in a product provides the best use of manufacturing methods requiring expensive dies and molds. For example, if a spherical metal tank is fabricated from identical segments having this new oblong shape, the segments are efficiently formed by stamping, forging, or hydroforming into a die and joined together by full penetration welding of abutting edges. Some segments may include attachments or bosses for pipes or other connections, which are not shown in Fig. 2A. The uniformity in width of the segments minimizes waste and improves economy in shipping. Segments for manufacture of a spherical vessel 15.24 m (50 feet) in diameter are readily transported by truck as they have a nearly constant width of less than 2.44 m (8 feet) and are approximately 6.1 m (20 feet) in length.

For many architectural and other applications, a designer may desire to construct a structure having the shape of a half sphere. Sphere 20 can be conveniently divided into two halves along great circle line 22. Fig. 2B illustrates a resulting half dome with a smooth bottom edge, along line 22, that is obtained by this division. The dome is rotated slightly in Fig. 2B compared to the top half of Fig. 2A. The same segment 30 of Fig. 2A is also labeled on Fig. 2B for comparison. Only four of the spherical rhombuses are cut through by line 22, two diagonally between near corners and two diagonally between far corners. In Fig. 2A, diagonal cutting of a rhombic face by line 22 is shown as a dashed line running between far corners, which forms partial segments 26, 27, 28, and 29. No waste results from the cuts when making a half dome because, as shown in Fig. 2A, cut-off segment portion 26 can be used to fill the space at segment portion 29. Similarly, in Fig. 2B, segment portions 24 and 25 are obtained by a single cut of one full segment. Thus, a half dome can be constructed by assembling 30 quadrilateral segments; four of the segments are cut and rearranged to fill the four triangular positions at the dome edge. Other fractional portions of a sphere besides the half dome are readily constructed with suitable cutting of some segments.

This spherical dome construction from oblong quadrilaterals is a substantial improvement over the existing methods, as the structure is remarkably efficient regarding energy and materials; furthermore, all of the segments are identical and of convenient shape, are very easily assembled, and provide superior strength with less weight. This simplest embodiment has broad-ranging applications, of which the following example is illustrative.

A spherical dome greenhouse or skylight is a useful application of the above assembly with oblong segments. The pieces are vacuum formed from clear plastic sheet and then trimmed to the segment shape. Very little material is wasted in trimming because the segments are close in form to the rectangle from which they are formed and cut. In a preferred embodiment of this unbraced dome construction, additional width and length are included in the segments and they are joined together by overlapping of edges, in a fashion described below with reference to Fig. 6. In an alternative embodiment, flanges are provided on the edges of segments, an example of which will be described below in connection with Figs 4A and 4B. Flanges and overlapping edges are joined by screws and nuts, by adhesive, or by other customary means along abutting or overlapping surfaces. Some segments preferably include modifications for ventilation (not shown).

A very inexpensive and lightweight greenhouse can be manufactured from 30 segments in this manner. For a dome 3.81 m (12.5 feet) in diameter, each segment measures less than 0.61 m (two feet) wide, is 1.52 m (five feet) long, and has 15.24 cm (six inches) of rise in the center due to its spherical curvature. The spherical form provides excellent stiffness for comparatively thin sheet, such as clear polycarbonate plastic in 1.0 mm (0.040 inch) thickness. The spherical structure has sufficient strength as to require no frame. The segments for a 3.81 m (12.5-foot) dome weigh approximately 1.0 kg (2.2 pounds) each, and a box of thirty segments for a half dome can be shipped by a common parcel service. Opaque or reflective segments (not shown) can be substituted for clear ones at any location as needed for light control. The overlapped and fastened segments are easily removed and changed to adjust for the changing seasons.

In the following Figs. 3A through 13, only a representative portion of each embodiment, such as a single rhombic unit, is shown. These figures do not accurately display the three-dimensional form, being schematic in character. It will aid the reader's understanding to consider that all of the figures except Fig. 13 utilize the same general symmetry properties of a triacontahedron as displayed in Figs. 2A and 2B. Further, while all assemblies include a means for joining parts, this is not shown in the figures except in Figs. 4A-6 where it is essential to the description of the embodiment.

Variations of the Basic Form—Ambiguity

Figs. 3A and 3B illustrate that an ambiguity exists concerning mirror images. Both Figs. 3A and 3B show the division of a spherical rhombic face as described above, in the simplest form of my improved segmentation, and together they illustrate a duality of possible segment shapes. The two figures show identical spherical rhombic faces that are convex upward (toward the reader) and with acute angles arranged vertically. Each figure shows a labeled segment with a first long side 40 near to parallel with a second long side 42, a first short side 44, and a second short side 46. The sides are connected in alternation, long to short, and then short to long, etc., to produce a segment with the approximate appearance of a parallelogram conformed to a sphere. The interior angles formed between the sides are first through fourth angles 48, 50, 52, and 54.

The pair of segments in either of these figures are identical. They have the same component parts, and in the same order for their figure. The same central cut has been made in the two figures, except that, in Fig. 3A the cut is to the left and in Fig. 3B the cut is to the right. Like parts are labeled with the same numbers in the two figures. In comparing segment 30 of Fig. 3A with segment 32 of Fig. 3B, it is evident that like parts have the same measure, but they are arranged in reverse order. As the faces are convex upward, the difference in order cannot be corrected by turning over one face. Thus the two illustrated segments are equal by every scalar measure, but are mirror images and therefore not identical. This ambiguity of solutions occurs for all divisions of a spherical rhombic face where the number of cuts inclined to the left is different than the number of cuts to the right. Of course, the two types of segment shape can be mixed pair-wise, within the practical construction of a single spherically-shaped structure. For clarity of presentation, illustrated embodiments subsequent to Fig. 3B are composed of segments conforming to the Fig. 3A type. This simplified presentation should not be construed as implying any preference of one type over the other, or to exclude mixing of types, when advantageous, from the scope of the invention.

The quadrilateral segment of Fig. 3A embodies my improved shell segmentation in its simplest form. The segment and its possible variations will be defined in greater detail by reference to an enlarged view in Fig. 3C, showing a spherical rhombic surface bounded by sides 34, 36, 38, and 42. For a spherical triacontahedron, the sides have an equal arc measure of 37.3774 degrees, the acute angles measure 72 degrees, and the obtuse angles measure 120 degrees. Segment 30 and its duplicate are formed as the surface is divided by line 40, a great circle arc generally parallel to sides 38 and 42. Division by line 40 creates short sides 44 and 46 and also angles 48 and 54 of segment 30. Thus, the sum of the short sides is 37.3774 degrees of arc, and the sum of angles 48 and 54 is 180 degrees. There is some ambiguity in the placement of line 40 and therefore also in the quantitative definition of the segment with regard to the newly formed sides and angles. This will be illustrated by three particular examples.

Variations of the Basic Form—Example 1

As a first example in illustrating variation in the position of the dividing line, consider the requirement that line 40 be chosen to be parallel with lines 38 and 42 on the sphere. Specifically, referring to Fig. 3C, line 40 is the perpendicular bisector of a line 41, which line is

perpendicular to both lines 38 and 42 and passes through center point P of the spherical rhombic surface. Line Wm is the right half of line 41 and is the maximum width of segment 30 for this example. Line Wm will have the same measure in all examples to be presented, being the perpendicular distance from the point P to side 42. Lines Wt and Wb depict the widths of the segment at its ends; they depend upon the particular selection for line 40, as do short sides 44 and 46 and interior angles 48 and 54. The calculated dimensions for the segment of this first example are presented in the first data column of Table 1 below. Referring to the table, the segment tapers slightly in width from the middle dimension Wm to the end widths Wt and Wb, with the top end being about 0.8 degree less in width than the bottom, and about 1.1 degrees less than the middle width. Due to the curvature of a spherical rhombus, first long side 40 is about 2.5 degrees longer than second long side 42. The length of line 40 also depends upon the particular way it is chosen.

Table 1.—Dimensions of Three Quadrilateral Segment Examples for a 60-Segment Division of a Sphere.

SEGMENT PART		EXAMPLE 1 Long Sides ‖	EXAMPLE 2 Short Sides =	EXAMPLE 3 Wt = Wb .
First Long Side	40	39.9293 deg	39.1908 deg	39.4903 deg
Second Long Side	42	37.3774	37.3774	37.3774
First Short Side	44	17.7818	18.6887	18.31095
Second Short Side	46	19.59555	18.6887	19.0664
First Angle	48	115.1713	112.86775	113.8379
Second Angle	50	72.0000	72.0000	72.0000
Third Angle	52	120.0000	120.0000	120.0000
Fourth Angle	54	64.8287	67.13225	66.1621
Top Width	Wt	16.8847	17.7427	17.3854
Middle Width	Wm	18.0000	18.0000	18.0000
Bottom Width	Wb	17.6698	17.1720	17.3854
Variation in Width		1.1153	0.9 approx.	0.6685

Variations of the Basic Form—Example 2

As a second example, line 40 is alternatively required to bisect sides 34 and 36 in Fig. 3C. Symmetry ensures that line 40 will pass through the center point P. For the construction of this example, Wm is not the maximum width of segment 30. The location of maximum width lies toward first short side 44 from point P and the width is slightly greater than Wm. Some dimensions have changed substantially from those of the first example. Changed dimensions are lines Wt and Wb, which are the widths of the segment at its ends, first long side 40, short sides 44 and 46, and interior angles 48 and 54. The calculated dimensions for this second segment example are presented in the second data column of Table 1. Referring to the table, the top width is now greater than the bottom width by about 0.6 degree, which is somewhat better than for Example 1 and in the opposite direction. Line 40 is now only about 1.8 degrees longer than line 42. The changed dimensions are the result of line 40 having rotated counterclockwise by 2.4479 degrees relative to the position of the first example.

Variations of the Basic Form—Example 3

As a third example in illustrating variation in the position of the dividing line, consider the requirement that line 40 be so chosen as to obtain equal widths at the ends of the segment. The resulting position of line 40 is found to be intermediate between the positions for Examples 1 and 2. The dimensions for this case have been determined, and are presented in the third data column of Table 1. Of course, the table now shows that end widths Wt and Wb have an equal measure of 17.3854 degrees. The place of maximum width is only very slightly above the location of point P, and the maximum width is 18.0539 degrees, only slightly more than W. The segment shape of this third example has excellent uniformity, varying in width over its length by only 0.6685 degrees.

All three of the above segment examples, as well as any other segment shapes within the range of variation of these examples, are useful in the design of a quadrilateral segment for practical assembly of a sphere or spherical dome.

Variations of the Basic Form—Generalization of the Principle

It is not essential that the sides of the surface displayed in Figs. 3A-C be great circle arcs. Any practical and useful construction that preserves the rotational symmetry of the parent polyhedral face will suffice. In the broadest view, any arbitrarily formed line on the sphere used to replace the great circle arcs in joining the four end points on the surface of Fig. 3C will possess the same symmetry properties. In particular, small deviations of the rhombic sides of Fig. 3C from great circles allows the construction of oblong quadrilaterals with even higher uniformity of width than is displayed in Table 1. Further, it is not necessary that the surface be spherical, as will be illustrated later with regard to Figs. 8, 9, 11A, and 11B. And, viewing line 40 in the broadest view, any arbitrarily formed line in the surface with the same center of symmetry as the surface will cut the surface into two identical parts. All practical constructions with the desired symmetry properties fall within the scope of this invention.

Joining and Sealing of Segments—Figs. 4A to 5B

In order to assemble the physical segments into a useful structure, a method of joining them must be provided. A variety of joining means may be utilized, such as welding of metal shells, or use of flanges, clevis joints, or overlapping joints with suitable fasteners. Two examples of overlapping joints will be shown: use of underlapping flanges in Figs. 4A-C, and use of splicing and offsetting elements in Figs. 5A and 5B. Overlapping of segment edges is a method that is especially effective in developing the full strength capability of a shell. A well-secured overlapping connection provides very efficient transfer of all types of loads, such as tension, compression, shear, and moments.

Accordingly, the second embodiment shown in Figs. 4A-C, which depicts a physical quadrilateral segment 60 of the order discussed above, further incorporates joining means into the segment as underlapping flanges and means for their attachment. In Fig. 4A, flanges 62, 64, and 66 extend from the edges of the basic shape that was illustrated in the previous figures. Flange 62 extends outwardly from the lower half of the first long side and flanges 64 and 66 extend from the short sides. The flanges are generally beveled at the corners to avoid interference with other flanges from adjoining parts. The flanges are made to follow the spherical form of the segment and have top surfaces recessed sufficiently to mate in alignment

with the undisplaced bottom surfaces of the overlapping edges of adjoining segments. This underlapping feature provides that the joints present minimum disruption on the exterior surface of the assembled structure.

Fig. 4B shows the segment in assembled relationship to adjoining edges or flanges. Dashed lines indicate hidden edges of underlapping flanges. Solid lines are the visible lines of overlap where exterior exposed surfaces of adjoining segments meet. Flange 62 is a part of segment 60 while flange 68 is the same part of the mating segment to the left, which is in a reversed position relative to segment 60. For the present joining method, the first long sides of the segments, shown as the left side of segment 60, must have half of their lengths provided with flanges so that they interlock in a mating fashion such as shown. While the first long sides are mated together in pairs, all other joining portions comprise short sides underlapped to second long sides. Such is the case for flanges 64 and 66, which are part of segment 60, and for flanges 56 and 58, which are respective corresponding parts of adjoining segments.

A fastener is used to hold a flange and mating overlapping edge together. Any fastening which is suitable for the material of the segments may be used, such as screws, bolts, adhesive, latching materials, stitching, brazing, or the like. Presently, for smaller applications, I prefer to use flange head screws with washers and nuts, in combination with a sealing adhesive, substantially as shown in cross-sectional view in Fig. 4C. Underlapping flange portion 64 of segment 60 is joined to overlapping edge 70 of an adjoining segment by fastener 72. Optionally, an adhesive may be applied between surfaces 74. Exposed surfaces 76 and 78 meet in perfect alignment at overlapping line 77. Assembly of shells by overlapping and fastening of segments is useful for many segment designs, using flanges as in Figs. 4A-C or in other readily evident configurations having equivalent functions. The underlapping flange method of joining will be presented in a second form in Figs. 5A and 5B.

In some situations, which are not illustrated, joints require sealing with a gasket or sealant material. The flanging method of this embodiment is applicable to a great many kinds and shapes of segments, and the present embodiment is meant to be illustrative of all such forms. The flanges may be regarded as integral to the segment shape in light of practical considerations such as shipping and segment manufacture. Thus, in some applications, the lines of overlap will not agree closely with the shape and dimensions of the first preferred embodiment without appurtenances previously presented.

My improved shell segmentation forms as thus far presented are well suited for applications to spheres and domes up to approximately 15.24 m (50 feet) in diameter. Such spherical forms can be manufactured using a great variety of materials, material thicknesses or combinations, and methods of manufacture. In particular, the second embodiment is especially well suited for making low-cost houses. For example, spherically-shaped domes may be made using segments that are formed from moldable materials, such as organic or inorganic fibers combined with resins or adhesives, in a single mold. The segments are laid up by hand in the mold and allowed to set. After curing or drying, the segments are treated with water repellents as necessary. When a sufficient number of segments are prepared, they are assembled rapidly using any of a variety of fasteners, as for example, screws combined with a waterproof adhesive, and secured to a foundation.

Assembly of Insulated Segments

Assembly of two spaced-apart shells of the types described above with insulation between creates a very-well-insulated building. Energy requirements for full-size dome houses are in some cases less than half that of a conventional house. Consider for example a half dome, 15.24 m (50 feet) in diameter, which is an appropriate size for use in the United States. The surface area of the dome is 365 m^2 (3927 square feet) and it encloses 176.5 m^2 (1,900 square feet) of floor area at ground level. An additional upper level of 102 m^2 (1,100 square feet) can be enclosed. Including the upper level, the house has 279 m^2 (3,000 square feet) of usable floor area and a floor-to-surface ratio of approximately 0.8, indicating a very energy-efficient house form. (Cf. the floor-to-surface ratio of approx. 0.5 for a typical conventional house.) In an especially convenient method of assembly of the present spaced-apart dome structure, the inner and outer shell segments are joined in a pair-wise fashion before the dome is assembled. The combination of spaced-apart shell segments and enclosed insulation forms an insulated segment. This insulated segment construction will now be illustrated for a dome comprising 1/60th-spherical segments, such that each segment occupies $\pi/15$ steradians of solid angle in the structure, or exactly 30 segments for a half dome.

Another preferred embodiment of my improved segmentation, having an insulated segment construction, is shown in Fig. 5A in plan form. Fig. 5B is a cross-sectional view taken

along section line 5B-5B of Fig. 5A, showing the interior construction of the segment. Fig. 5B shows insulated segment 80 formed from inner shell segment 84 and outer shell segment 86, which are joined together in spaced-apart relationship by short splicing-offset element 87 (shown in Fig. 5A) and long splicing-offset element 88. Elements 87 and 88 are of uniformly C-shaped cross-section, of a form suitable for manufacture by a variety of methods, such as bending, molding, or extrusion and post-forming. The splicing-offset elements have dual functions. They first serve to hold the paired shell segments together and, at the time of assembly of the dome, they serve as underlapping flanges in a manner similar to that in Figs. 4A-C. Insulation 90 placed between joined shell segments 84 and 86 is shown schematically in the figures by dashed cross-hatching in Fig. 5A and as a sinuous line in Fig. 5B.

Long splicing-offset element 88, shown at the right side of Fig. 5B, in combination with affixed shell edges, forms a long male connector. Similarly, a short male connector is formed by short splicing-offset element 87 in combination with affixed shell edges (not shown in Fig. 5B). During construction of the dome, these male connectors are inserted into mating female connector portions of adjacent insulated segments. The left-most part of Fig. 5B shows female connector portion 92 of insulated segment 80. It is formed by edge 94 of outer shell segment 86, edge 96 of inner shell segment 84, and spacer 98. Each segment has two female connector portions, labeled 92 and 93 in Fig. 5A. Spacer 98 and splicing-offset elements 87 and 88 hold the shell segment edges in correct spaced-apart relationship for joining the female connector portions to male connector portions of the adjoining segments. As the insulated shell segments are joined together, margins of the individual segment insulations merge to form a continuous layer of insulation between the inner and outer shells. The location and margins of the interior insulation is illustrated in Fig. 5A by the bounds of the dashed cross-hatching.

An additional useful feature is illustrated in the insulated shell segment of Fig. 5A. The top right corner of the segment is shown removed to leave a small void 99. In joining of segments to form a dome, the voids of five segments join together to form an interstitial aperture. In a half dome, four apertures are formed in the shell in this way. The apertures are closed by insertion of interstitial elements. These elements can be blank closures, or selected to perform various desired functions for the structure, such as a window or skylight for admission of light, ventilator units, or a pass-through for plumbing vents or a furnace flue. The shape and size of voids 99 may be chosen to suit the designer's purposes, allowing insertion of interstitial

elements of virtually any size and shape. Examples of shapes for interstitial elements are polygons, spherical polygons, circular or spherical elements, or even "bubble-shaped" insertions with spheroidal form.

Additional preferred embodiments illustrated in Figs. 6 through 11B will be directed to higher orders of division of the triacontahedron face, either as divisions of a sphere or of polyhedra. Each succeeding embodiment is intended to serve as an additional illustration of the practical usefulness of an oblong segment for manufacture or construction of generally spherical forms such as spheres, large polyhedra, geodesic domes, and the like.

More Than 60 Spherical Segments—Figs. 6 and 7

In the above embodiments, a spherical rhombus is divided into two oblong segments. In general, a spherical rhombic face may be divided into M rows and N columns by any number of lines running generally parallel to its sides. The number of quadrilateral segments formed on the face is M x N, and the whole sphere is divided into P = 30 x M x N segments for the case of a spherical triacontahedron. A compact notation can be used for the order of division by representing the integers M and N as a number pair, (M,N). In the previous embodiments, M + N = 3 and there is just one dividing line. These can have either two rows and one column (2,1) or one row and two columns (1,2). As previously discussed, the two forms are not the same, but are mirror images. This duality extends through all cases of M not equal to N and thus can be obtained in the remaining embodiments, but will not be illustrated. Only examples having fewer rows than columns in analogy to Figs. 2A, 2B, 3A, and 3C are presented, but this should not be construed to limit the scope of my invention.

Fig. 6 illustrates a (1,4) division of a spherical rhombus and Fig. 7 illustrates a (3,12) division. The geometric segments cannot be identical for these cases with M + N > 3. However, uniformly-spaced dividing lines distribute the variations so that differences are small. Typically, the required segments vary in length by approximately 5% and in width by approximately 4%. These small variations and the constancy of curvature of a spherical segment suggest that physical segments which are identical, of a constant width, and which exceed the required geometric width and length by approximately 8% to 10%, may be assembled by overlapping to form spherical shells. Such segments of uniform crosswise section have many advantages. They can be formed at low cost by extrusion of materials such

as thermoforming plastics, or by other longitudinal forming methods of suitable materials, such as roll-forming. Spherical curvature is then added to the segments by post-forming. The slightly increased weight of uniform-width segments is more than offset by lower cost. Variations in overlap are used during assembly to control the assembled dimensions. Variations in overlap are not shown in the figures due to the difficulty of correctly rendering curved forms. Overlap of segments must be precisely controlled during assembly to accomplish the correct spherical shape. Methods for overlap control include providing pre-punched holes along the sides, or use of chord gages. A chord gage is a straight beam connected across a specified chord of the shell during assembly to insure the correct shell curvature and segment placement.

Fig. 6 shows a one-thirtieth part, or one spherical rhombic face, of a spherical triacontahedron. Rhombic face 100 is shown assembled from four segments of uniform crosswise section, such as segment 102, joined together by overlapping of edges. Short-dashed lines show hidden underlapped edges along long sides of segments. Long-dashed lines across the short sides of the segments are intended to show the amount by which these sides are to be underlapped in joining to the adjacent segments (not shown). Optionally, the underlapped ends may be offset in a fashion similar to that shown in Fig. 4C. The amount of overlap shown in the drawing is approximately 10%. As an example of this embodiment, 30.5 cm (12-inch) wide segments 1:5 m (five feet) long assembled as shown form a dome approximately 3.81 m (12.5 feet) in diameter. For this example, a useful overlap of segment long sides can vary from about 2.54 cm (one inch) at the center of a segment to about 3.81 cm (1.5 inches) near the ends. Considering larger segments, a 30.5 m (100-foot) diameter half dome can be formed from segments that are approximately 2.44 m (8 feet) wide and 12.2 m (40 feet) long. Such segments have an ideal size and shape for transportation by truck

Fig. 7 illustrates a still higher-order division of a spherical rhombus 110, having 36 segments per 1/30th part of a sphere. Each segment, such as segment 108, is approximately 14 degrees in arc length and 3.2 degrees of arc wide. This preferred (3,12) segmentation of a spherical shell is well suited for construction of large architectural domes having a bracing understructure (not shown). For example, segments 2.44 m (8 feet) wide and 11.28 m (37 feet) long are used to construct a quarter dome 76.2 m (250 feet) in diameter and 22.86 m (75 feet) high. A quarter dome is one-half radius high; in this 76.2 m (250-foot) diameter structure, the

radius of curvature is 45.72 m (150 feet). Although this illustration is the last of the type to be shown, it should be understood that there is no limit, in principle, to the order of division that can be used. Joining of segments may be accomplished by a variety of means (not shown) at the discretion of the designer.

Uniform Crosswise Section—Manufacturing Methods

Small segments such as shown above, when joined by overlapping, are well suited to be manufactured by extrusion, roll forming, slitting, stamping, shearing, or the like. The designer has additional options for simplifying manufacture of segments with constant cross-section, especially for materials with a degree of elasticity such as plastics. They are: (1) Formation of both ends of the segments at once by a single oblique cut with a shear at the time of extrusion. (2) Elimination of the post-forming step to achieve spherical curvature in the segments. In this second method, segments are given a crosswise cylindrical curvature of a somewhat smaller radius than the intended spherical radius. The segments are flexed to the desired shape at the time of assembly. (3) Further, elimination of cylindrical curl, allowing some narrow segments to be simply cut from flat material. For the segments of Fig. 7, for example, only 3.2 degrees of flexure are required to obtain the desired crosswise curvature.

Fig. 7 completes the presentation of embodiments of my improved segmentation based upon straight-forward division of a spherical rhombic face into rows and columns by evenly-spaced lines. All segments thus formed have the appearance of a parallelogram conformed to a sphere. Segments having the general appearance of a parallelogram, with nearly equal opposite sides and opposite angles, are discernible in many forms of my improved segmentation, which general form constitutes a new and distinct characteristic of this advance over the prior art.

Segmentation of Polyhedra—Figs. 8 to 11B

Numerous additional embodiments of my improved shell segmentation are obtained by its application to polyhedral structures such as geodesic domes. As before, the improved segmentation yields oblong segments which are identical, or identical in groups, and are especially well suited for efficient and economical construction of many-faced polyhedral shells of generally spherical form. Four preferred embodiments are illustrated in Figs. 8-11B. While

the illustrations of Figs. 8, 9, 10, and 11B are schematic only and presented as if they were flat, they actually have considerable convex form. An effort is made to show the convex form of a portion of a polyhedron in Fig. 11A. The lines shown in these figures represent edges between angularly-joined polygonal faces, and line junctions are vertices of the polyhedral surfaces, with the single exception of the lighter lines of Fig.10. All of the embodiments of Figs. 8-11B are regarded as higher-order divisions in terms of the number of polyhedral faces. However, the polyhedral structures of Figs. 8, 9, 11A, and 11B will be shown to be divisible into identical oblong clusters of faces which are similar in form to the previously described quadrilateral segments, and also in accordance with the present invention. The embodiments of Fig. 10 and the alternative segment shape of Figs. 11A and 11B each comprise multiple groups of identical segments. The beneficial use of interstitial elements is illustrated again, for polyhedra, in the alternative shape of Figs. 11A and 11B.

The diagram of Fig. 8 illustrates a one-thirtieth portion of a polyhedron having 240 triangular faces. In the figure, eight triangular faces are shown, face 112 being representative. All lines in the figure are edges of the polyhedron and all junctions of lines are vertices. Lines 114 and 116 joining at vertex 118 divide the eight triangular faces into two equal parts, 120 and 130, which are identical oblong segments. Segment 120 has a first long side composed of lines 114 and 116, a second long side composed of lines 122 and 124, a first short side 126, and a second short side 128. Segment 130 has a first long side composed of lines 114 and 116, a second long side composed of lines 132 and 134, a first short side 136, and a second short side 138. The polyhedral, substantially quadrilateral segments of Fig. 8 are of (1,2) order. Thus, 30 segments join to make a polyhedral half dome with 120 triangular faces.

Alternatively, the segments can be constructed by marking vertex points at the corners and centers of long sides of a spherical segment of (1,2) order, and then joining vertices by lines to form the triangular-faced segment. The segments of Fig. 8 have two sides which are jointed, or not straight. In perfect similarity to segments of a sphere having curved sides, so segments of a polyhedron in general have jointed sides. These segments have many of the benefits of the corresponding spherical segments, such as all being identical and of a convenient shape for shipping and rapid assembly. The segments optionally incorporate a modular bracing structure with the skin, and also a means of joining the segments. For some materials, it may be more convenient to form a polyhedral segment than a spherical segment. For example, metal sheet

can be bent along the edge lines with an inexpensive brake. This segment is not as strong as the equivalent spherically-formed stamping, but is simpler to manufacture in a small shop.

The polyhedral segments of Fig. 9 follow the same segmentation plan as in Fig. 8. In this case, the two illustrated (1,2) segments have eight rhombic faces each, and 30 segments join to make a polyhedral half dome having 240 rhombic faces. For example, segment 140 comprising the right half of the figure includes rhombic face 142. Fig. 10 presents a more complex segmentation of a polyhedron, illustrating a division into (1,3) segments. The full polyhedron has 350 hexagons with twelve pentagons interspersed among them in the positions of dodecahedron face centers. The number of segments used to form the polyhedron is 90 and, therefore, two kinds of segments are required. Referring to Fig. 10, each rhombic unit contains one middle segment 150 and two side segments 160. Unlike the two previous polyhedral examples of Figs. 8 and 9, the dividing lines forming the segments do cut through some of the faces of the polyhedron. Corners 154 of the outside segments, which include segment interior angles measuring 72 degrees, join together in groups of five to form the pentagons of the structure. Corners 156 of the outside segments, which include segment interior angles having a measure of 120 degrees, join together in threes to form a portion of the hexagons. Half hexagons 158 and 162 on the ends of middle segment 150 combine with half hexagons of the adjoining side segments (not shown) which are in the same position as half hexagon 164 of side segment 160. In this instance and many others that could be presented, the assembled segments combine in a cooperative fashion to generate patterns that continue beyond their borders, and leave no incomplete faces.

Illustration of the Generalized Principle

Fig. 11A shows a portion of a polyhedron that is extraordinary for its natural beauty and utility. The polyhedron comprises twelve pentagons and 120 hexagons. The pentagons are shaded in the illustration so that they may be clearly distinguished. This geometric figure is particularly useful as a pattern for small to medium-sized domes, up to about 30.5 m (100 feet) in diameter. It is also amenable to division into 60 identical oblong segments without a need to cut through the hexagonal faces. Two oblong segments, each comprising a 1/60th part of the polyhedron, are displayed in Fig. 11B. Segment 170 is shown to consist of triangular tip 172, first hexagon 174, and second hexagon 176, joined together in angular alignment. This

assembly of two segments fits the description provided in the previous section, under "Generalization of the Principle." One can see that the two segments are identical and appear together to have the same rotational symmetry as the triacontahedron face which they emulate. The triangular tips of the segments join five together to form the pentagons, such as pentagon 178, of the polyhedral surface. The triangular tip 172 of segment 170 can be removed to yield a corner interstitial void as previously described in connection with Fig. 5A. Then, a half dome assembly comprises 30 identical segments, each having two hexagons, and six pentagonal interstitial elements. In this alternative segmentation of the polyhedron of Figs. 11A and 11B, smaller identical oblong segments are achieved by forming an interstitial element as a second segment shape, all of which elements comprise a second group of identical segments.

Again, as illustrated by the examples above of the segmentation of polyhedra, the use of identical, oblong segments has been shown to provide a substantial advance in the art, allowing simpler and less expensive construction of shells having a generally spherical form. While further practical examples can be given of polyhedral domes with fewer or more divisions and with less or greater complexity, the four embodiments presented in Figs. 8-11B are sufficient for illustrating the basic principles of segmentation of polyhedral structures into oblong, substantially quadrilateral or similar forms.

Other Unclassified Embodiments—Figs. 12 and 13

Some highly symmetrical segmentations of the sphere have oblong elements of substantially uniform width and have great commercial value, but are not conveniently represented as divisions of a spherical rhombic polyhedron. The embodiment of Fig. 12 is an example. Fig. 12 depicts a one-tenth part, approximately, of a complete spherical shell comprising 60 oblong spherical quadrilateral faces (six shown) and twelve interstitial elements of regular spherical pentagon form (three shown). Segment 180 is one of the oblong faces and spherical pentagon 198 is one of the interstitial elements. The spherical pentagons are referred to as interstitial because each one resides in an interstice positioned centrally between ten oblong segments. The spherical pentagons can be replaced by other interstitial figures of differing shape and materials by making a slight alteration of the edges of the quadrilaterals. The symmetry of the segmentation allows an arbitrary size for the interstitial elements, with the quadrilaterals adjusting their form to fill the space between. In the limit of vanishing interstitial

elements, the geometric figure becomes the same as in Figs. 2A-3C. In the limit of vanishing quadrilaterals, the figure becomes a spherical dodecahedron.

Fig. 12 shows pentagons and quadrilaterals of approximately equal width. As in the first embodiment, quadrilateral 180 has a first long side 182, a second long side 184, a first short side 186, a second short side 188, and first through fourth angles 190, 192, 194, and 196. The segment is narrower and longer than in Figs. 2A-3C, but angles are comparable, with angle 192 measuring a little less than 72 degrees, angle 194 equal to 120 degrees, and the sum of angle 190 and angle 196 equaling 180 degrees. This segment also has nearly constant width, with variation comparable to that of the first embodiment. A continuum of quadrilateral dimensions, from those of the first embodiment to beyond this example, are achievable and within the scope of the invention.

The quadrilaterals shown have a nearly constant width of approximately 15 degrees as compared to 18 degrees for the quadrilaterals of Figs. 2A-3C. The shell area comprises approximately 95% quadrilaterals and only 5% pentagons. Using this model, a half dome of 8.53 m (28 feet) diameter can be made from 30 oblong segments 1.22 m (4 feet) wide and approximately 3.20 m (10.5 feet) long. Six interstitial pentagons 1.22 m (4 feet) wide are also required to complete the structure. Components for the entire shell can be carried in a pickup truck. At the other size extreme for my shell segmentation are large public and industrial dome structures. As a final example, a design model comprising rectangular elements will be presented that is especially well suited for construction of very large domes of low aspect.

Rectangular Structures

A spherically-curved dome roof of generally square appearance is shown in plan view in the embodiment of Fig. 13. This shell 200 is formed by joining together four groups of segments, each group together having the approximate form of a spherical square with sides having a length of 40 degrees of arc in this example. Square group 202 includes oblong segment 204, for example. The resulting assembly has a comparatively low height-to-width ratio, or aspect, of approximately 0.18 at the midsection. Due to the spherical curvature, the corners dip down much lower than the midsection edges, giving an aspect at the corners of approximately 0.34. This is the only embodiment to be presented in which the segmentation cannot be used to form a complete sphere or polyhedron. This fact lends more freedom in the

segmentation plan, allowing reallocation of space so that all physical segments of the assembly are spherical rectangles of substantially identical size with least variation in overlap of edges.

A consequence of such reallocation is that the outside lines of each square group lie in planes that deviate approximately 15 degrees from the corresponding great circle arcs. As an example, a square building according to this model, built from 128 rectangular segments of dimensions 2.44 m (8 feet) by 4.88 m (16 feet), is 30.5 m (100 feet) wide and 10.7 m (35 feet) high with the corners of the roof directly supported at the ground. The structure presents sweeping arches on its four sides, each arch reaching a height of 4.57 m (15 feet). Note that there is no particular scale requirement for this design. For example, by quadrupling the number of segments and halving their curvature, segments of the same size will form a pavilion 61 m (200 feet) wide and 21.3 m (70 feet) high. By selectively removing rows of segments from Fig. 13, this design can be used to model substantially spherical rectangular structures of varying proportions, and such structures are suitable for joining together in further rectangular arrays. While this embodiment lacks the elegance displayed in earlier figures, it makes up for the lack by its great design flexibility, usefulness and efficiency of construction.

Conclusion, Ramifications, and Scope

Thus, it is evident from the above illustrative examples that shells having a generally spherical form and comprising a plurality of substantially identical, conveniently-shaped, oblong segments, with other optional elements, provide many advantages over the prior art. These include (1) savings in energy and material resources, (2) simplified construction, (3) improved manufacture, packing and shipping, (4) reduced labor and materials, (5) improved weight and strength, and (6) greater adaptability to varying materials and manufacturing processes.

My spherical and polyhedral shells with improved segmentation include all such shells as above described, in which the predominant segment shape is oblong and essentially uniform in width. This beneficial shape can take a great many forms, for example, a portion of a sphere, or a group of polygons joined at their edges. The segment shapes are most often derived from rhombic faces by dividing them into smaller faces, at least two, and grouping the smaller faces into oblong parts running generally parallel with sides of the original rhombic face. If the parent face is a spherical rhombus, then the component segments are spherical quadrilaterals of

substantially uniform width. The method of derivation of the segments from rhombic faces and their nearly constant width suggest that all such segment shapes have the general appearance of a parallelogram, whether they are spherical or polygonal. However, no completely satisfactory term can describe them, since infinitely many shapes are possible. The shapes can include other parts, such as cutouts or voids, flanges, or other special features.

Therefore, the present shell segmentation is a substantial improvement.

While the above description contains many specifics and examples, numerous ramifications unfold from the great many choices for its implementation, including the following variations.

1. The shell may be a whole sphere or polyhedron, half of the whole figure, such as a half dome, or any other greater or lesser portion.

2. The shell may have a planar edge, as from a whole figure cut by a plane, or may be a portion of a sphere or polyhedron not obtained by cutting with a plane, but which is useful for the intended application. An example is a roof structure with a pleasantly-varying edge, obtained by using only whole oblong rectangular segments.

3. The shell may be joined to other elements such as rectangular or cylindrical forms, or to other similar shells; it may include specialized parts for holding windows, doors, or the like; and it may be joined to an underlying framework, or a second inner, spaced-apart shell.

4. The shell may beneficially include a second or additional segment shapes besides the predominant oblong shape, and may include partial segments as well. In particular, interstitial elements may be used to fill apertures formed between groups of other segments.

5. Segment shapes may be derived by division of polyhedra other than the spherical or ordinary triacontahedron, such as the rhombic dodecahedron, regular dodecahedron, octahedron, cube, tetrahedron, other irregular polyhedra, and spherical analogs of the above.

6. Segment shapes may be derived by division of a small group of polygons, spherical polygons, or other similar figures which can not be extended to form a complete polyhedron or spherical polyhedron.

7. Possible variations in some sides and angles of spherical segments have been described. Additionally, any of the many segment shapes can be further generalized by substituting faces and edges of arbitrary form, provided only that the substituted forms preserve the essential symmetry properties of the segment.

8. For each of the many possible segment shapes, they may be either of two mirror-image types, or may be of both types within a single shell.

9. Segments may be made of a wide variety of soft or rigid materials, such as metals, plastics, composites, fabrics, and so forth, and joined by various means such as welding or brazing, adhesives, riveting and bolting of overlapping edges or flanges, stitching and latching. A segment may include a combination of materials, for example, a segment may have an outer layer of one material, an inner layer of a second material, and optionally other layers between.

10. Segments having different materials or structures, to serve specific purposes, may be combined together in a single shell. For example, one segment or interstitial element may include glass or clear plastic to serve as a window or skylight. Another segment may include a doorway and associated features. Interstitial elements may also be useful for stress-distribution and as structural attachments, for vents, or for pass-throughs. Interstitial elements may be chosen from a variety of shapes, such as flat polygons, circular disks, and spherical or spheroidal shapes, for example.

The details of the various illustrated embodiments and further ramifications have been provided for exemplary purposes only. Numerous changes in the mathematical and structural details provided may be made without departing from the scope of the following claims and their equivalents.